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Momentum Dependent Photon-Vector Meson Coupling and Parity Violating effects in $B \to X_s \ell^+ \ell^-$ *

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Abstract

We examine the leptonic forward-backward and polarization (left-right) asymmetries in the dileptonic $B \to X_s \ell^+ \ell^-$ decay when the momentum dependence of ψ and $\psi' - \gamma$ conversion strength is taken into account. The results indicate only a small shift in the asymmetry distributions.

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In a recent work [1], we have shown that, when the momentum dependence of photon-vector meson coupling is taken into account, the "resonance to nonresonance" interference in the dileptonic invariant mass distribution of $B \to X_s \ell^+ \ell^-$ is substantially reduced. This momentum dependence, which is necessary to explain the data on ψ leptonic width and photoproduction simultaneously [2], is also believed to suppress the long-distance (LD) contributions to $b \to s\gamma$ decay [3,4]. The significance of this investigation is due to the fact that the CKM favored intermediate $\psi(NS)$ vector mesons contribute to $b \to s$ transitions by conversion to real($b \to s\gamma$ decay) or virtual($b \to s\ell^+\ell^-$ decay) photons [5-8]. These resonance contributions, in fact, dominate the total dileptonic decay rates. Therefore, to probe the contributing short-distance (SD) operators for, among other things, signals of "new physics", one has to take a careful account of the LD interference.

In this letter, we examine the effect of the momentum dependence of ψ and $\psi' - \gamma$ conversion strength on the parity-violating aspects of the dileptonic rare B-decays. The lepton pair forward-backward asymmetry [9] and polarization asymmetry distributions [10] have been proposed as possible venues to probe the SD contributions. Our results indicate that, unlike the invariant mass spectrum, these asymmetry distributions are not altered significantly due to a momentum dependent photon-vector meson coupling.

We start with the low energy effective Lagrangian for $b \to s\ell^+\ell^-$:

$$L_{eff} = \frac{G_F}{\sqrt{2}} \left(\frac{\alpha}{4\pi s_W^2} \right) V_{ts}^* V_{tb} (A \bar{s} L_\mu b \bar{\ell} L^\mu \ell + B \bar{s} L_\mu b \bar{\ell} R^\mu \ell + 2m_b C \bar{s} T_\mu b \bar{\ell} \gamma^\mu \ell), \tag{1}$$

where

$$L_\mu = \gamma_\mu (1 - \gamma_5), \quad R_\mu = \gamma_\mu (1 + \gamma_5),$$

and

$$T_\mu = -i\sigma_{\mu\nu}(1 + \gamma_5) q^\nu / q^2.$$

V_{ij} are the Cabibbo-Kobayashi-Maskawa matrix elements, $s_W^2 = sin^2\theta_W \approx 0.23$ (θ_W is the weak angle), G_F is the Fermi constant and q is the total momentum of the final $\ell^+\ell^-$ pair.

The SD parts of A and B, denoted by $A^{\rm SD}$ and $B^{\rm SD}$, arise from W box diagrams and

penguin diagrams with Z gauge boson and photon coupled to $\ell^+\ell^-$ pair. For $m_t = 180GeV$, $m_b = 4.5GeV$ and $\Lambda_{QCD} = 100MeV$ we obtain [11]

$$
\begin{aligned}
A^{\rm SD} &= 2.020, \\
B^{\rm SD} &= -0.173, \\
C &= -0.146.
\end{aligned}
\tag{2}
$$

The LD part of A and B coefficients receive contributions from charm quark loop ($c\bar{c}$ continuum), and ψ and ψ' resonances.

$$
A^{\rm LD} = B^{\rm LD} = -s_W^2 \left(3C_1(m_b) + C_2(m_b)\right) \left(\tau^{\rm cont} + \tau^{\rm res}\right) \tag{3}
$$

The combination of the Wilson coefficients in (3) is assigned a value

$$
|3C_1(m_b) + C_2(m_b)| = 0.72
$$

which fits the data on the semi-inclusive $B \to X_s \psi$ [3]. The $c\bar{c}$ continuum contribution is obtained from the electromagnetic penguin diagrams [12]

$$
\tau^{\rm cont} = g\left(\frac{m_c}{m_b}\ ,\ z\right) \tag{4}
$$

where $z = q^2/m_b^2$ and

$$
g(y\ ,\ z) = \begin{cases} -\left[\frac{4}{9}\ln(y^2) - \frac{8}{27} - \frac{16}{9}\frac{y^2}{z} + \frac{2}{9}\sqrt{1-\frac{4y^2}{z}}\left(2+\frac{4y^2}{z}\right)\left(\ln\frac{|1+\sqrt{1-\frac{4y^2}{z}}|}{|1-\sqrt{1-\frac{4y^2}{z}}|} + i\pi\right)\right] & z \geq 4y^2 \\ -\left[\frac{4}{9}\ln(y^2) - \frac{8}{27} - \frac{16}{9}\frac{y^2}{z} + \frac{4}{9}\sqrt{\frac{4y^2}{z}-1}\left(2+\frac{4y^2}{z}\right) arctan\frac{1}{\sqrt{\frac{4y^2}{z}-1}}\right] & z \leq 4y^2 \end{cases} \tag{5}
$$

The resonance contributions from ψ and ψ' can be incorporated by using a Breit-Wigner form for the resonance propagator [6,7]:

$$
\tau^{\rm res} = \frac{16\pi^2}{9}\left(\frac{f_\psi^2(q^2)/m_\psi^2}{m_\psi^2 - q^2 - im_\psi\Gamma_\psi} +\ (\psi \to \psi')\right)e^{i\phi} \tag{6}
$$

The relative phase ϕ that determines the sign between $\tau^{\rm cont}$ and $\tau^{\rm res}$ is chosen to be zero due to unitarity constraint [13].

From (6) we observe that $\tau^{\rm res}$ depends quadratically on $f_V(q^2)(V=\psi\ ,\ \psi'\ )$ defined as:

$$< 0|\bar{c}\gamma_\mu c|V(q) >= f_V(q^2)\epsilon_\mu \tag{7}$$

where ϵ_μ is the polarization vector of the vector meson V. It has been pointed out recently that in the context of Vector Meson Dominance, data on photoproduction of ψ indicates a large suppression of $f_\psi(0)$ compare to $f_\psi(m_\psi^2)$ [3]. This has been confirmed independently in Ref [4] by constraining the dominant LD contribution to $s\to d\gamma$ using the present upper bound on the $\Omega^-\to\Xi^-\gamma$ decay rate. In fact, as we mentioned earlier, it is argued that this large suppression results in a much smaller LD contribution to $b\to s\gamma$ transition.

In the dileptonic rare B-decays, however, $f_V(q^2)$ is normally replaced with the decay constant $f_V(m_V^2)$ obtained from the leptonic width of ψ and ψ':

$$\Gamma(V\to\ell^+\ell^-)=\frac{16\pi\alpha^2}{27m_V^3}f_V^2(m_V^2)$$

The invariant mass spectrum obtained this way, is dominated by the resonance interference for a broad range of q^2, as already noted in the literature [8,9]. However, as it was indicated in Ref [1], this spectrum which (using (1)) is written as:

$$\begin{aligned}\frac{1}{\Gamma(B\to X_c e\bar{\nu})}\frac{d\Gamma}{dz}(B\to X_s\ell^+\ell^-) = & \left(\frac{\alpha}{4\pi s_W^2}\right)^2\frac{2}{f(m_c/m_b)}\frac{|V_{ts}^*V_{tb}|^2}{|V_{cb}|^2}(1-z)^2\\ & \times\ \Big((|A|^2+|B|^2)(1+2z)+2|C|^2(1+2/z)\\ & +6Re[(A+B)^*C]\Big)\end{aligned} \tag{8}$$

where

$$f(x)=1-8x^2+8x^6-x^8-24x^4ln(x),$$

shows a significant reduction in "resonance to nonresonance" interference when the momentum dependence of f_V (or equivalently, $\psi-\gamma$ transition) is taken into account. As depicted in fig. 1, at $q^2/m_b^2\approx 0.3$, for instance, the resonance interference amounts to around 2% of the differential branching ratio as compared to 20% in the case where fixed $f_V(m_V^2)$ is used.

In this work, we focus on the parity violating asymmetry distributions in the inclusive rare decay $B\to X_s\ell^+\ell^-$ when a momentum dependent $f_V(q^2)(V=\psi\ ,\ \psi')$ is inserted in

$\tau^{\rm res}$ (we assume that the same suppression occurs for ψ'). This momentum dependence is derived in Ref [2] based on the intermediate quark and antiquark state:

$$f_V(q^2) = f_V(0)\left(1 + \frac{q^2}{c_V}\left[d_V - h(q^2)\right]\right) \tag{9}$$

where $c_\psi = 0.54$, $c_{\psi'} = 0.77$ and $d_\psi = d_{\psi'} = 0.043$. $h(q^2)$ is obtained from a dispersion relation involving the imaginary part of the quark-loop diagram:

$$h(q^2) = \frac{1}{16\pi^2 r}\left\{-4 - \frac{20r}{3} + 4(1+2r)\sqrt{1-\frac{1}{r}}arctan\frac{1}{\sqrt{1-\frac{1}{r}}}\right\} \tag{10}$$

with $r = q^2/4m_q^2$ for $0 \leq q^2 \leq 4m_q^2$. m_q is the effective quark mass and assuming that the vector mesons are weakly bound systems of a quark and an antiquark, we take $m_q \approx m_V/2$. As a result, eqn (9), defined for $0 \leq q^2 \leq m_V^2$, is an interpolation of f_V from the experimental data on $f_V(0)$(from photoproduction) and $f_V(m_V^2)$(from leptonic width) based on quark-loop diagram. We assume $f_V(q^2) = f_V(m_V^2)$ for $q^2 > m_V^2$ mainly due to the fact that the behavior of $\psi - \gamma$ conversion strength is not clear in this region.

The forward-backward asymmetry distribution is defined as:

$$A^{FB}(z) = \frac{\int_0^1 dw d^2BR/dwdz - \int_{-1}^0 dw d^2BR/dwdz}{\int_0^1 dw d^2BR/dwdz + \int_{-1}^0 dw d^2BR/dwdz}, \tag{11}$$

where $w = cos\theta$ with θ being the angle between the momentum of the B meson (or the outgoing s quark) and that of ℓ^+ in the center of mass frame of the dileptons. Using the effective Lagrangian (1) one obtains a simple form for this asymmetry in the $m_s = 0$ limit [9]:

$$\begin{aligned} A^{FB}(z) &= \frac{3}{2}\frac{\left((|A|^2 - |B|^2)z + 2Re[(A-B)^*C]\right)}{\left((|A|^2 + |B|^2)(1+2z) + 2|C|^2(1+2/z) + 6Re[(A+B)^*C]\right)} \\ &= \frac{3}{2}\frac{\left(A^{SD} - B^{SD}\right)\left((A^{SD} - B^{SD} + 2ReL)z + 2C\right)}{\left((|A|^2 + |B|^2)(1+2z) + 2|C|^2(1+2/z) + 6Re[(A+B)^*C]\right)}, \end{aligned} \tag{12}$$

Where $L = A^{LD} = B^{LD}$. The forward-backward asymmetry distribution is shown in fig. 2, where the distributions without τ^{res} and the one with constant ψ, $\psi' - \gamma$ conversion strength are also depicted. We observe that the momentum dependence of the photon-vector meson coupling results in a small shift in the asymmetry distribution. In fact, as it

is demonstrated, away from the peaks, the exclusion of the resonances does not change this distribution significantly.

On the other hand, from (12) we observe that $A^{FB}(z)$ is proportional to $A^{SD} - B^{SD} = A - B$, the coefficient of the leptonic axial vector current, which does not receive QCD corrections and its numerical value (2.193 for $m_t = 180 GeV$) can be determined accurately [12]. However, the second factor in the numerator of (12) is sensitive to QCD corrections through $A^{SD} + B^{SD} + 2ReL = Re(A + B)$, and C, the coefficient of the magnetic moment operator. The latter enters the rare B-decay $B \to X_s \gamma$.

At this point, we would like to remark that the determination of the invariant mass $0.1 < z_\circ < 0.2$, where the forward-backward asymmetry vanishes, can serve to determine $A^{SD} + B^{SD} \approx -2C/z_\circ$, when C is known from other channels eg. $B \to X_s\gamma$. This is due to the fact that one can safely ignore the LD effects in this region (see fig. 2).

Next, we turn to the lepton polarization asymmetry distribution which is defined as:

$$P(z) = \frac{dBR/dz|_{\lambda=-1} - dBR/dz|_{\lambda=+1}}{dBR/dz|_{\lambda=-1} + dBR/dz|_{\lambda=+1}}, \tag{13}$$

with $\lambda = -1$ ($\lambda = +1$) corresponding to the case where the spin polarization is anti-parallel (parallel) to the direction of the ℓ^- momentum. In the limit of massless leptons, this simplifies to left-right asymmetry as left and right-handed leptons do not mix in this limit. Therefore, one obtains $dBR/dz|_{\lambda=-1}$ ($dBR/dz|_{\lambda=+1}$) from (8) by setting the coefficients of the right-handed (left-handed) leptonic current equal to zero. In this way, one gets the following expression for the left-right asymmetry:

$$\begin{aligned} A^{LR}(z) &= \frac{\Big((|A|^2 - |B|^2)(1+2z) + 6Re[(A-B)^*C]\Big)}{\Big((|A|^2 + |B|^2)(1+2z) + 2|C|^2(1+2/z) + 6Re[(A+B)^*C]\Big)} \\ &= \frac{\Big(A^{SD} - B^{SD}\Big)\Big((A^{SD} - B^{SD} + 2ReL)(1+2z) + 6C\Big)}{\Big((|A|^2 + |B|^2)(1+2z) + 2|C|^2(1+2/z) + 6Re[(A+B)^*C]\Big)}, \end{aligned} \tag{14}$$

We notice that A^{LR}, like A^{FB}, is proportional to a purely SD coefficient ie., $A^{SD} - B^{SD}$. The asymmetry $A^{LR}(z)$ is shown in fig. 3 for cases where i) momentum dependence of $f_V(q^2)$ is taken into account, ii) $f_V(q^2)$ is replaced with fixed $f_V(m_V^2)$ and iii) τ^{res} is excluded.

We observe that the asymmetry distribution, away from the peaks, receives only small interference (the deviation from the curve with τ^{res} set to zero) from the resonances. For example, at $z = q^2/m_b^2 = 0.3$, this interference amounts only to 2% of the asymmetry at this invariant mass.

In conclusion, we investigated the effect of a momentum dependent photon-vector meson coupling on various distributions of the dileptonic rare B-decay $B \to X_s \ell^+ \ell^-$. The effect is more significant in the invariant mass spectrum (fig. 1), resulting only in a small shift in the forward-backward (fig. 2) and left-right (fig. 3) asymmetry distributions. In light of these results, we believe that one can get reliable complementary information about SD physics from these distributions.

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Figure Caption

Figure 1: The dileptonic invariant mass spectrum for the decay $b \to s\ell^+\ell^-$. The thin, dotted and bold lines correspond to the spectrum without resonances, with resonances but constant $V-\gamma$ conversion strength and with resonances having momentum dependent $V-\gamma$ transition respectively. For $q^2 > m_{\psi'}^2$, where the latter two curves coincide, only the dotted curve is shown.

Figure 2: The forward-backward asymmetry distribution for the decay $b \to s\ell^+\ell^-$. The thin, dotted and bold lines correspond to the asymmetry without resonances, with resonances but constant $V-\gamma$ conversion strength and with resonances having momentum dependent $V-\gamma$ transition respectively. For larger values of z, where the latter two curves coincide, only the dotted curve is shown.

Figure 3: The polarization (left-right) asymmetry distribution for the decay $b \to s\ell^+\ell^-$. The thin, dotted and bold lines correspond to the asymmetry without resonances, with resonances but constant $V-\gamma$ conversion strength and with resonances having momentum dependent $V-\gamma$ transition respectively. For larger values of z, where the latter two curves coincide, only the dotted curve is shown.

Figure 1

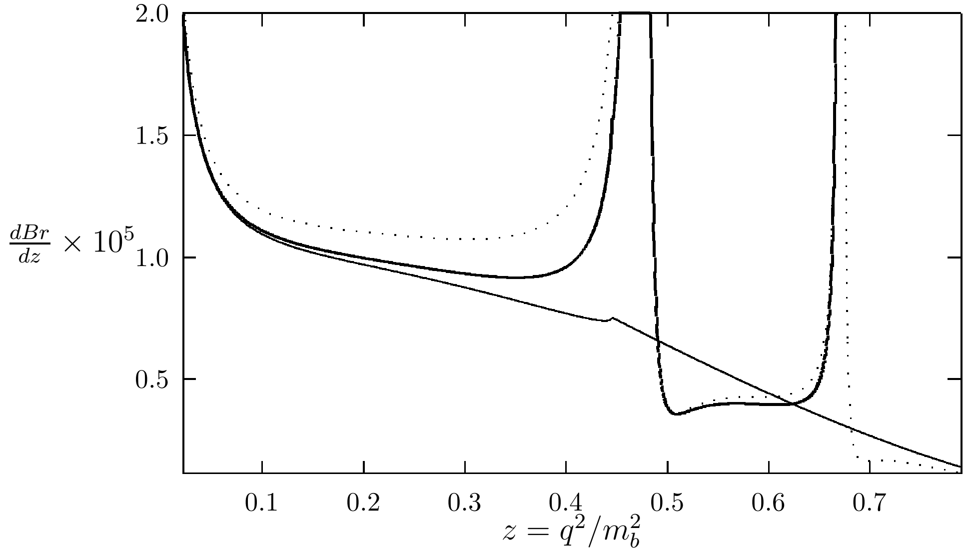

Figure 2

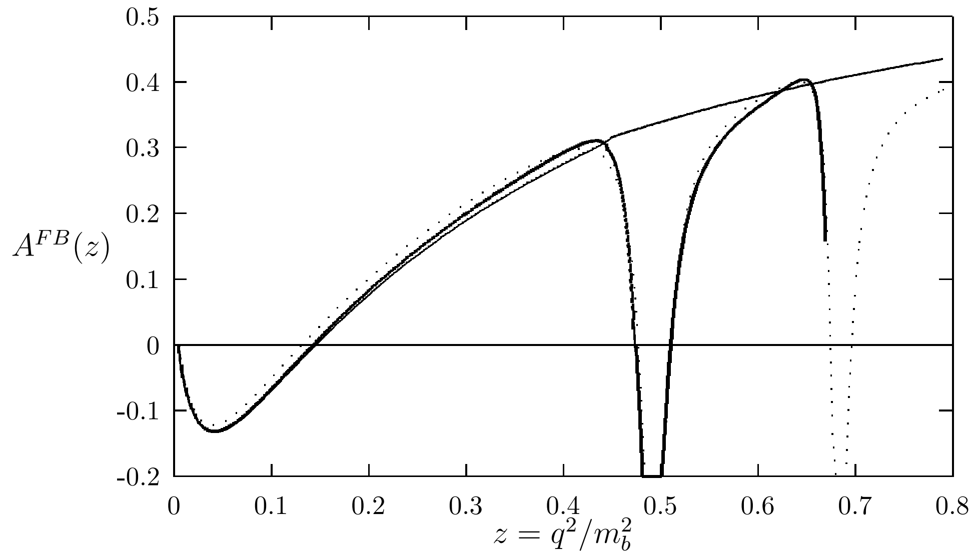

Figure 3